

Contribution of public and private higher education to innovation and economic growth

VERSÃO FINAL APÓS DEFESA

Carolina Basílio Neves

Dissertação para obtenção do Grau de Mestre em
Economia
(2º ciclo de estudos)

Orientador: Prof. Doutor Pedro Cunha Neves
Co-orientadora: Prof. Doutora Elena Sochirca Neves

outubro de 2021

Dedictory

I want to dedicate this work to my parents, my sister, and my boyfriend, who have always been by my side, on the good and not so good days that this whole path has given me. Thank you very much to everyone who attends me in every step of the way, for encouraging me to achieve my dreams and, above all, for always believing, day after day, in me! Thank you very much!

Acknowledgments

As the development of this dissertation was one of the most important and challenging periods of my life, due to all its demands and sacrifices, I would like to express my sincere thanks to all the people who, directly or indirectly, made its completion possible. This dream could not have come true without the support of several people who were essential to achieve all these objectives. To all who were by my side and witnessed each victory and each defeat, a huge thank you!

My first thanks go to my parents and sister for the motivation, affection, and financial availability that they always gave me to achieve my academic path. I probably would not be here without their support.

Secondly, I thank my supervisor, Professor Pedro Neves and my co-supervisor, Professor Elena Sochirca, for all the support, help, attention, and availability. A huge and heartfelt thank you for having transmitted me some of your knowledge, for the availability, commitment, and strength that you transmitted to me throughout this journey so that I would not give up on this cause even in the most complicated moments.

I must also thank all my colleagues and friends for the strength, motivation, and companionship they have given me during this journey. Finally, I have to thank my boyfriend for his love, support, care, strength and motivation and for having supported me in all the stages of this journey.

Resumo

Aprender é um dos pontos chave para o crescimento, tanto do ser humano como do próprio país. Este último depende quer da tecnologia quer da ciência que utiliza. O nexu I&D – crescimento económico tem recebido cada vez mais atenção na literatura. Contudo, existem ainda perguntas sem respostas no que toca à diferenciação imposta aos trabalhadores dos vários setores quando se referem ao tipo de educação que estes recebem ao nível do ensino superior. O presente estudo desenvolve um modelo de crescimento económico endógeno com recurso a *Knowledge Driven* e I&D horizontal com o objetivo de examinar a diferença entre o ensino superior público e privado e a sua contribuição para o crescimento económico de um país.

Palavras-chave

Crescimento Económico; I&D; Inovação Horizontal; Universidades privadas e públicas; produtividade

Abstract

Learning is one of the basic keys to growth, both for the human being and the country itself. The latter depends on both the technology and the science it uses. The R&D - economic growth nexus has received increasing attention in the literature. However, there are still unanswered questions regarding the differentiation imposed on workers in the various sectors when referring to the type of education they receive at the higher education level. The present study develops an Endogenous Economic Growth model with Knowledge Driven horizontal R&D with the aim of examining the difference between public and private higher education and its contribution to the economic growth of a country.

Keywords

Economic Growth; R&D; Horizontal innovation; Private and Public Universities; Productivity

Table of contents

1. Introduction.....	1
2. Literature Review.....	4
3. The model	10
4. Public versus Private higher education: Possible scenarios for analysis.....	19
5. Results and Discussion	23
6. Conclusion	28

Figures list

Figure 1 - Costs and Debt levels for students, considering the type of institution, in USA	19
Figure 2 - The net present value of future earnings for students, in \$, considering the type of institution, in USA.....	19

Tables list

Table 1 - Top 20 universities in the USA, considering potential early and mid-career salary of alumni.....	20
Table 2 - Employer reputation of the top 15 universities in the world, according to the QS World University 2021 Ranking.....	21
Table 3 - Variables description.....	23
Table 4 – Scenarios for analysis, for the period 1960-2019, using the calibration method	24
Table 5 - Results of each scenario, using equation xviii and the variables present in table 3, for the period 1960-2019.....	29

Acronyms list

I&D	Investigação e Desenvolvimento
R&D	Research and Development
USA	United States of America
TFP	Total Factor Productivity
ROI	Return on investment
EU	European Union
GDP	Gross Domestic Product
ERASMUS	European Region Action Scheme for the Mobility of University Students
ASEAN	Association of Southeast Asian Nations

1. Introduction

Nowadays, it is essential to understand what factors influence the differences between regions of the globe and more importantly, understand ways to minimise them (Ezcurra and Rodríguez-Pose, 2009; Iammarino et al., 2019); one such factor is education (Baten and Hippe, 2018). Both the Human Capital Theory and the New Growth Economy Theory define education as a core determinant of economic growth and development at national and regional levels (Romer, 1990; Mankiw et al., 1992; Batabyal and Nijkamp, 2013). Considering the various levels of education, higher education is the most successful in providing competitive skills for the global economy and keeping up with technological developments (Faggian and McCann, 2009). Any country expects this type of education to be one of the factors that contributes most to economic growth and, with this emphasis, can explain the economic differences between the various regions (Sianesi and Van Reenen, 2003).

The Research and development (R&D) - Economic Growth nexus has received increasing attention in the literature. In the last three decades, several theoretical and empirical models have shown that both R&D and education of individuals are two of the main drivers of economic growth (Bronzini and Piselli, 2009; Forman and Zeebroeck, 2012). The difference in terms of economic growth among countries can be explained by the productivity of the human capital factor, obtained from the creation or innovation of products or processes that involves spending on R&D. Lichtenberg (1992) states that one of the strategies that ensure the process of economic growth and technical progress is R&D expenditure. We can go back to 1942 with Schumpeter to understand the importance of R&D in economic growth. Schumpeter (1942) argued that the creation of knowledge through R&D is essential to ensure productive efficiency and sustained economic growth, referring to "creative destruction" as a driver of sustained economic growth in sectors (Verstraete, 2002). Romer (1994), based on the work of Schumpeter (1942), showed through an endogenous growth model that R&D contributes significantly to increasing productivity and, consequently, the economic growth of nations.

The term "knowledge economy" is defined as an economic system of which growth results mostly from the intellectual capabilities of individuals (Sepehrdoust and Zamani, 2015) and, consequently, from the accelerated pace of technological and/or scientific advancement. A wide network of economists' states that this type of economy is the latest phase of global economic restructuring, accompanied by technological and competitive innovations, determinants of R&D, higher education institutions, laboratories, among other educational entities.

Among the most recent studies of endogenous growth models with R&D, Chiu et al. (2017) analyse the formation of ideas in such a model, concluding that a country's productivity increases with innovation. Gamlath and Lahiri (2018) theoretically analyse the degree of substitutability between public and private expenditure on education and its impact on long-term economic growth. In their model, human capital in adulthood is determined by both education received in childhood and human capital of parents and, institutional and cultural factors influence the parents' decision to choose for private rather than public education (Gamlath and Lahiri, 2018), that is higher substitutability implies higher human and physical capital stocks per capita in steady state and a faster transition to it, leading to higher economic growth rates. Authors such as Glomm & Ravikumar (1992), Epple and Romano (1996a), de la Croix and Doepke (2004) and Goldhaber (1999) have considered such expenditure as substitutable. On the other hand, Epple and Romano (1996b) and Gouveia (1997) concluded that a public investment in education by the state is complemented by additional private spending. The existing literature mentions the role of higher education institutions in the economic growth process and suggests that the advances and setbacks of R&D are important determinants of the evolution of regional economic growth (Denti, 2010). However, it does not distinguish whether the fact that the institution is public or private influences the economic performance of countries differently. As Glomm (1997) and Das (2007) have analysed it is essential to understand the role of public expenditure on education and consequent human capital accumulation (Sochirca et al., 2017).

In general, there are no studies that combine the deployment of the two types of workers in the economic growth process, i.e., those with private higher education or those with public higher education; a model of economic growth that includes this difference is needed. Thus, the research question of this study is: What is the contribution of the type of education of the worker at the higher education level in the economic growth process of a country?

Our study brings something new to the literature as, building on an Endogenous Economic Growth with horizontal Knowledge Driven R&D, it accounts for differences between workers' higher education (public versus private) and examines how these differences affect the process of innovation and economic growth. As the influence of such differences in higher education is a topic that has not been addressed in related literature, it becomes important given the differentiation that is currently imposed on students from public versus private universities.

The objectives of the study are to clarify the differences regarding the return to productivity of a worker trained in a public or a private institution, determining the influence of these differences on the economic growth and innovation processes.

The model under analysis in this study falls into the category of Semi-endogenous models. It is considered that in the economy there are three sectors: a final goods sector, an intermediate goods sector and an R&D sector. Both the final goods sector and the R&D sector employ workers with public higher education and workers with private higher education and each type of worker has different productivity levels. We aim to determine how, in the context of the model, these different productivity levels influence the innovation process and the rate of economic growth. The results, using a calibration method for three different scenarios, will provide policy implications concerning the funding and provision of both types of higher education.

The present study is structured as follow. In Section 2 we present the literature on endogenous growth models with R&D and the importance of making a distinction between both public and private higher education in the process of economic growth. In Section 3 describes the endogenous growth models with horizontal R&D. In Section 4 we present real facts behind the possible three scenarios analysed in section 5. In Section 5 we present the results of the model using a calibration method to those scenarios and a discussion of the results. Section 6 concludes and presents policy implications and prospects for future work.

2. Literature Review

The Solow's (1956) model was considered the starting point for the formation of the neoclassical theories of growth with which we are familiar today, since the attempts of Harrod (1939) and Domar (1946) had produced negative results as regards the stability of capitalist systems. One of the main conclusions of Solow's (1956) neoclassical growth model is that long-run economic growth rates are exogenously determined because they do not depend on the levels of savings and the production function and, in the long run, the economy converges to the steady state, whatever the initial capital-labour ratio. Another conclusion of the Solow base model is that, in this model, output per capita, capital per capita, consumption per capita and savings per capita are constant in the long run. In the neoclassical model with technical progress, the growth rates of output and consumption depend on the rate of technical progress, which is exogenous in the model, i.e., it is not explained by the dynamics of the model, the economic factors and the behaviour of individuals and firms (Solow, 1956). Solow (1956) eliminated the instability of the growth rate in Harrod's (1939) model and ignored the variables that stabilised it according to him, disregarding the short-term hypotheses implicit in the investment function and in the level of technical progress that he mentioned. Solow (1956), by introducing a one-dimensional model where he only considered the evolution of the capital-labour ratio, also ignored the relationship that the economic growth rate had with the profit rate variable. Thus, three main criticisms of the Solow (1956) model are presented: the exogenous growth rate, the differences in the growth rates of the various countries and the convergence in growth rates. The work of Solow (1956) and Swan (1956) gave rise to the so-called modern analysis of economic growth.

However, Solow (1956), according to the new Growth Theory, makes no mention of technological change and its effect on the growth of Total Factor Productivity (TFP). His 1956 article is considered a reference in the economic growth literature because it concluded that the growth rate of per capita product in an economy, once the steady state is reached, will only be sustainable if there is technical progress (Solow, 1957). In turn, Romer (1990) explained that this technological change or development results from the profit maximization of agents that invest in research in exchange for monopoly profits and that the existing knowledge results in new ideas and innovations, whereby the growth of per capita output is an increasing function of the amount of human capital in an economy and, consequently, of its technological progress. Thus, Romer (1990) concluded that the difference in the stock of human capital existing between economies is an element of differentiation of their economic growth rates (Moura and Cruz, 2013). The term "Knowledge Economy" is defined as an economic system in which growth results mostly from the intellectual capabilities of its individuals and, consequently, from

the accelerated pace of technological and scientific advancement (Sepehrdoust and Zamani, 2015).

Drucker (1966) introduced the foundations of the Knowledge Economy, describing the difference between the manual worker and the knowledge worker, stating that the first works with his hands producing goods or services and, the second works with his head producing ideas, knowledge and/or information (Walter et al., 2014). The new Growth Theory has technological change as its main driver which in turn affects TFP growth. This new theory was introduced by Romer (1990) with the Nobel Prize in Economics, with the perception of new ideas as a non-rival input and output of investment in innovation, allowing endogenizing R&D in long-term economic growth models.

In the same line of thought as Solow's model (1957), the approach of Moura and Cruz (2013) with an endogenous growth model highlights the role of technological progress as a driver of a country's economic growth by analysing the main determinants of the evolution of technical progress.

The level of prosperity of many so-called rich countries is often attributed to the British Industrial Revolution that provided remarkable technological changes, improvements at the social level, higher incomes, and a higher level of economic growth as well (Acemoglu and Robinson, 2012). The industrialisation phenomenon increased the importance of human capital in production processes, complementing with physical capital. Over time, there was a need for a higher and more comprehensive level of education as the level of industrialisation increased and industrial work became more demanding. Thus, policy reforms provided populations with higher levels of education, in some cases reaching higher education.

R&D is usually known to produce innovations that improve product development and production processes, contributing to the economic growth of countries (Nair et al., 2020). The R&D - Economic Growth nexus has received increasing attention in the literature. In the last three decades, several theoretical and empirical models have shown that both R&D and training of individuals are two of the drivers of economic growth (Bronzini and Piselli, 2009; Forman and Zeebroeck, 2012). In developed countries, the human capital factor is observed in greater abundance, namely the availability of infrastructure and technologies that improve the education and training of individuals, while in less developed countries this abundance does not occur. Sometimes, the difference between these countries can be explained by the productivity of this factor, derived from the creation or innovation of products or production processes which implies R&D spending. Lichtenberg (1992) even states that one of the strategies that ensure the process of economic growth and technical progress is R&D spending.

We can go back to 1942 with Schumpeter to understand the importance of R&D in economic growth. Schumpeter (1942) argued that the creation of knowledge through R&D is essential to guarantee productive efficiency and sustained economic growth. Schumpeter refers to "creative destruction", a term identified by the author himself in 1942 (Schumpeter, 1942), derived from the work of Karl Marx as an economic theory of innovation and business cycles. According to this theory, the "creative destruction" encompasses the process of industry change that drives the economic structure of that sector, destroying the older one and creating a new one (Schumpeter, 1942). This author mentions that, the pioneer of innovation creates an opportunity to profit, enhancing competitiveness in the sector in which it is inserted and promoting the process of sustained economic growth (Verstraete, 2002). Romer (1994), based on the work of Schumpeter (1942), showed through an endogenous growth model that R&D contributes significantly to increase productivity and, consequently, the economic growth of nations. In simplified form, there are four generations of R&D-based economic growth models: i) endogenous growth models with unintended scale effects or 1st generation models that use the assumption that innovations are aggregated to the production function (Bond-Smith, 2019) (e. g. Romer, 1986; Grossman and Helpman, 1991; Jones, 1995a); ii) semi-endogenous or 2nd generation growth models that only assume linearity in population growth (Bond-Smith, 2019), i.e. the sustained growth rate is not affected by the increase in innovations but seen as an equilibrium reaction of profit maximizing agents (e. g. Jones 1995b; Kortum, 1997; Segerstrom, 1998); iii) Schumpeterian models of endogenous growth without scale effects which assume that the knowledge production function is only linear at the industry level (Bond-Smith, 2019) and that technological development occurs at the firm level (e. g. Young, 1998; Dinopoulos and Thompson, 1998; Peretto, 1998; Howitt, 1999); finally iv) the 4G models that contemplate exponentially increasing returns and a corresponding population growth (e. g. Peretto, 2018; Bond-Smith, 2019). Note that the theories and respective Endogenous Growth models are grouped by generations considering the linearity criterion (Bond-Smith, 2019). The main criticism mentioned by Bond-Smith (2019) is directed to the first and last models mentioned because the assumption is applied in the function of innovations. In the literature on Models of endogenous growth with R&D, two distinct innovation processes are also considered: vertical (e.g., Romer, 1990, Jones, 1995b) and horizontal (e.g., Segerstrom et al., 1990, Grossman and Helpman, 1991; Aghion and Howitt, 1992). The first consists in the introduction of a quality in an already existing product and/or production process, while the second consists in the creation of a new product and/or production process, which may create an opportunity to open a new sector. There are models by authors such as Dinopoulos and Thompson (1998), Peretto (1998), Young

(1998) and Howitt (1999) based on both innovation processes. The introduction of horizontal rather than vertical innovation allowed the elimination of scale effects in early R&D-based models of economic growth (Cozzi and Spinesi, 2006). Nair et al. (2020) have shown that the richest countries are at the forefront of R&D investments, ensuring the continuous growth of the global innovation value chain.

Taking population wealth into account, Perotti (1996) emphasises that greater inequality is associated with lower levels of human capital accumulation and, therefore, lower levels of economic growth. De La Croix and Doepke (2009) and Arcalean and Schiopu (2016) analysed the relationship between income inequality and public intervention in education considering consumer preferences concluding that, in poor economies, higher inequality decreases public expenditure per student, increasing the number of students in public schools; they found the opposite in rich economies.

Among the most recent studies of endogenous growth models with R&D, one of note is Chiu et al. (2017) who analysed the formation of ideas in such a model, concluding that a country's productivity increases with innovation. Gamlath and Lahiri (2018) theoretically analyse the degree of substitutability between public and private expenditure on education and its impact on long-term economic growth. These authors suggest an overlapping generations model, where an individual's human capital in adulthood is determined by both the education he receives in childhood and the human capital of his parents. In their study, institutional and cultural factors influence the parents' decision to opt for private rather than public education, thus influencing the economic growth process (Gamlath and Lahiri, 2018), i.e., higher substitutability implies higher human and physical capital stocks per capita in steady state and a faster transition to steady state, leading to higher economic growth rates. Authors such as Glomm & Ravikumar (1992), Epple and Romano (1996a), de la Croix and Doepke (2004) and Goldhaber (1999) have considered such expenditure as substitutable. Epple and Romano (1996b) and Gouveia (1997) concluded that a public investment in education by the state is complemented by additional private spending.

Considering the type of taxation applied by the state, Sochirca et al. (2017) combine elements of endogenous growth theory and new political economy. These authors assume that the accumulation of human capital is the engine of endogenous growth and show that, the policy effects of this accumulation play a crucial role, and can have negative effects on economic growth when a distortionary taxation policy is chosen (e.g. Alesina and Rodrik, 1992; Perotti, 1996) or, positive effects when it comes to economic structures with public investments in education (e.g. Perotti, 1992; Alesina and Rodrik, 1992; Persson and Tabellini, 1992).

The existing literature mentions the role of higher education institutions in the economic growth process and suggests that R&D advances and setbacks are determinants in the evolution of regional economic growth (Denti, 2010). However, it does not distinguish whether the fact that the institution is public or private influences the economic performance of countries differently. According to Denti (2010), higher education institutions play an important role in local and/or regional innovation and it is necessary to understand the impact that these institutions have on the economic growth of economies.

While Afonso et al. (2019a) distinguish two types of workers assigned to the final product, those with lower qualifications and those with high qualifications, the model under analysis in this study elaborates a distinction at the level of education in higher education of workers assigned to the final product and to the R&D sector, and it can be public or private, something new for the literature on the topic.

There is a large literature analysing the long-term economic growth outcomes associated with public funding of education (Blankenau and Nicole, 2004; Dissou et al., 2016; Sochirca et al., 2017). However, none of the studies mention education at the higher education level. As Glomm (1997) and Das (2007) have analysed it is essential to understand the role of public expenditure on education and consequent human capital accumulation (Sochirca et al., 2017). However, it is also necessary for macroeconomic models to consider the impacts of private expenditure that families spend on their children's education and the State's expenditure on it and, in turn, the consequences that these investments and financing have on wages, on the productivity of workers in each of the sectors, R&D and final goods, and, above all, on the growth rate of each country's per capita product, when the model allows it.

The present study becomes particularly interesting due to the fact that it is something new to the literature where, through theoretical work using a model framed within the literature of Endogenous Economic Growth with horizontal Knowledge Driven R&D, the aim is to clarify the difference between public and private training that is acquired in higher education and its influence on the economic growth of a country.

As the influence of the type of education that is acquired in higher education is a topic that has not been addressed in any other study in the literature according to the perspective adopted, it becomes relatively important given the differentiation that is imposed today on students from public versus private universities. It is necessary to combine several economic mechanisms and understand their interaction in the economic growth of a country (Sayer, 2000). Establishing the relationship between certain variables, namely the rate of economic growth, the percentage of workers in the R&D sector and in the final goods sector and the percentage of workers with public

education and with Private education is fundamental in the context of Endogenous Economic Growth.

3. The model

The model built in this study falls into the category of semi-endogenous growth models, considering the configuration given to the knowledge production function which will depend, in this case, on the type of higher education training, public or private, that researchers/innovators experience.

Based on endogenous growth models with horizontal R&D (e.g., Romer, 1990; Jones, 1995), three sectors are considered: a final goods sector, an intermediate goods sector and an R&D sector.

In the final goods sector, the final good is produced in a context of perfect competition, using labour and capital as inputs. Since $H_f = H_p + H_s$, the production function will be as follows:

$$Y = AH_{fP}^{\beta} H_{fS}^{\varepsilon} X^{\alpha}$$

where A represents the level of productivity in the production of Y , X the level of physical capital (usually represented by K), H_{fP} the total number of workers in the final goods sector with private tertiary education, H_{fS} the total number of workers in the final goods sector with public tertiary education, and β and ε are the shares of workers with private and public tertiary education, respectively, in global income.

In general, H corresponds to the total number of workers in the economy, H_f the total number of workers in the final goods sector, H_N to the total number of workers in the R&D sector, H_S to the total number of workers with public education and H_P to the total number of workers with Private education, such that:

$$H = H_{fP} + H_{fS} + H_{NP} + H_{NS}$$

Note that $\beta + \varepsilon = (1 - \alpha)$. In this model, $X = (\sum_{i=1}^{\infty} X_j^{\alpha})^{\frac{1}{\alpha}}$ hence, $Y = AH_{fP}^{\beta} H_{fS}^{\varepsilon} \sum_{j=1}^N X_j^{\alpha}$.

This production function can also be rewritten as follows given that $X_j = X$:

$$Y = AH_{fP}^{\beta} H_{fS}^{\varepsilon} \int_{j=1}^N X_j^{\alpha} dj \Leftrightarrow Y = AH_{fP}^{\beta} H_{fS}^{\varepsilon} (NX)^{\alpha} N^{1-\alpha}$$

Given that the price of the final good tends towards 1 and, the price of intermediate good j being p_j , firms maximise the present value of all future profits.

$$\max_{H_{fS}, H_{fP}, X_j} \int_0^{\infty} \left(p_Y Y - w_{fS} H_{fS} - w_{fP} H_{fP} - \int_0^N p_j X_j dj \right) dt$$

w_{fS} and w_{fP} represent, respectively, the wages of workers in the final goods sector with education at the public and private higher education level. This maximization is assumed to incorporate no intertemporal type elements, and therefore:

$$\max_{H_{fS}, H_{fP}, X_j} p_Y Y - w_{fS} H_{fS} - w_{fP} H_{fP} - \int_0^N p_j X_j dj$$

$$\Leftrightarrow \max_{H_{fS}, H_{fP}, X_j} AH_{fP}^\beta H_{fS}^\varepsilon \int_0^N X_j^\alpha dj - w_{fS} H_{fS} - w_{fP} H_{fP} - \int_0^N p_j X_j dj \quad (\mathbf{i})$$

Let us now look at the two first-order conditions that the model entails:

- 1) For each type of worker, the point where marginal productivity equals wages are the point up to which firms employ labour:

$$\begin{aligned} \frac{d(\mathbf{i})}{dH_{fS}} &= 0 & \frac{d(\mathbf{i})}{dH_{fP}} &= 0 \\ \Leftrightarrow \varepsilon(H_{fS})^{\varepsilon-1} AH_{fP}^\beta \int_0^N X_j dj - w_{fS} &= 0 & \Leftrightarrow \beta(H_{fP})^{\beta-1} AH_{fS}^\varepsilon \int_0^N X_j dj - w_{fP} &= 0 \\ \Leftrightarrow \varepsilon \frac{AH_{fP}^\beta H_{fS}^\varepsilon \int_0^N X_j dj}{H_{fS}} - w_{fS} &= 0 & \Leftrightarrow \beta \frac{AH_{fP}^\beta H_{fS}^\varepsilon \int_0^N X_j dj}{H_{fP}} - w_{fP} &= 0 \\ \Leftrightarrow w_{fS} &= \varepsilon \frac{Y}{H_{fS}} \quad (\mathbf{ii}) & \Leftrightarrow w_{fP} &= \beta \frac{Y}{H_{fP}} \quad (\mathbf{iii}) \end{aligned}$$

- 2) The productivity of each of the capital goods equals its price:

$$\begin{aligned} \frac{d \left(\max_{H_S, H_P, X_j} AH_{fP}^\beta H_{fS}^\varepsilon \int_0^N X_j^\alpha dj - w_S H_{fS} - w_P H_{fP} - \int_0^N p_j X_j dj \right)}{dX_j} &= 0 \\ \Leftrightarrow \alpha \frac{Y}{NX_j} &= p_j \end{aligned}$$

Given the first order conditions, $X_j = H_{fP}^{\frac{\beta}{1-\alpha}} H_{fS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}}$ and the price-demand elasticity is $\frac{-1}{1-\alpha}$.

In the model under analysis, there are several firms producing the final good and several linked to the R&D sector, so $Y_i = AH_{fP_i}^\beta H_{fS_i}^\varepsilon \sum_{j=1}^N X_{ji}^\alpha$ or $Y_i = AH_{fP_i}^\beta H_{fS_i}^\varepsilon \int_{j=1}^N X_{ji}^\alpha dj$.

For firms operating in the intermediate goods sector, there is a trade-off between constraints on the use of existing ideas/prototypes and rewards to R&D activity. With Knowledge driven specialisation labour is input into the R&D sector and each firm will have at each t a net benefit stream equal $(p_j - 1)X_j$ where $X_j = H_{fP}^{\frac{\beta}{1-\alpha}} H_{fS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}}$.

Note that the previous expression is the demand for the intermediate good X_j by the firms producing the final good. Thus, the innovators' maximization problem for deciding the price of the invented intermediate good at time s is, according to the results of Afonso et al. (2019b):

$$\max_{p_j(t)} \int_s^\infty \exp[-\bar{r}(t-s)] \pi_j(t) dt \Leftrightarrow \max_{p_j(t)} \int_s^\infty \exp[-\bar{r}(t-s)] (p_j - 1) X_j dt$$

$$\Leftrightarrow \max_{p_j(t)} \int_s^\infty \exp[-\bar{r}(t-s)] \left[(p_j - 1) H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}} \right] dt \quad (\text{iv})$$

The average interest rate between time s and t will be $\bar{r}(s, t) = \frac{1}{t-s} \int_s^t r(w) dw$ and the discount factor if r equals the interest rate constant is $\exp r(t-s)$. The maximization performed in equation iv is equivalent to the maximization of current net benefits at each point in time. So:

$$\begin{aligned} \max_{p_j} \pi_t &= \max_{p_j} (p_j - 1) X_j = \max_{p_j} (p_j - 1) H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}} \\ &\Leftrightarrow \frac{d \left[(p_j - 1) H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}} \right]}{dp_j} = 0 \\ &\Leftrightarrow \frac{d \left[H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-\alpha}{1-\alpha}} - H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}} \right]}{dp_j} = 0 \\ &\Leftrightarrow \frac{-\alpha}{1-\alpha} H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-1}{1-\alpha}} + \frac{1}{1-\alpha} H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} p_j^{\frac{-2+\alpha}{1-\alpha}} = 0 \\ &\Leftrightarrow H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} \left[\frac{-\alpha}{1-\alpha} p_j^{\frac{-1}{1-\alpha}} + \frac{1}{1-\alpha} p_j^{\frac{-2+\alpha}{1-\alpha}} \right] = 0 \\ &\Leftrightarrow \frac{-\alpha}{1-\alpha} p_j^{\frac{-1}{1-\alpha}} + \frac{1}{1-\alpha} p_j^{\frac{-2+\alpha}{1-\alpha}} = 0 \\ &\Leftrightarrow p_j^{\frac{-1}{1-\alpha}} \left[\frac{-\alpha}{1-\alpha} + \frac{1}{1-\alpha} p_j^{\frac{\alpha-1}{1-\alpha}} \right] = 0 \\ &\Leftrightarrow \frac{-\alpha}{1-\alpha} + \frac{1}{1-\alpha} p_j^{\frac{\alpha-1}{1-\alpha}} = 0 \Leftrightarrow \frac{1}{1-\alpha} p_j^{\frac{-(1-\alpha)}{1-\alpha}} = \frac{\alpha}{1-\alpha} \\ &\Leftrightarrow p_j^{-1} = \alpha \Leftrightarrow p_j = \frac{1}{\alpha} > 1 \end{aligned}$$

Thus, $p_j = \frac{1}{\alpha} > 1, \forall j, \forall t$ gives innovators the possibility to overcome R&D costs.

Given the previous result, we obtain $X = X_j = H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} = H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{(1-\alpha)}} \quad (\text{v})$.

Substituting equation v into the initial production function:

$$\begin{aligned} Y &= A H_{fP}^\beta H_{fS}^\varepsilon \sum_{j=1}^N X_j^\alpha = A H_{fP}^\beta H_{fS}^\varepsilon \sum_{j=1}^N \left[H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{(1-\alpha)}} \right]^\alpha \\ &\Leftrightarrow Y = A H_{fP}^\beta H_{fS}^\varepsilon N H_{jP}^{\frac{\beta\alpha}{1-\alpha}} H_{jS}^{\frac{\varepsilon\alpha}{1-\alpha}} A^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{2\alpha}{(1-\alpha)}} \end{aligned}$$

Since $H_{jP} = H_{fP}$ and $H_{jS} = H_{fS}$, the expression of Y will be $Y = A^{\frac{1}{1-\alpha}} N H_{fP}^{\frac{\beta}{(1-\alpha)}} H_{fS}^{\frac{\varepsilon}{(1-\alpha)}} \alpha^{\frac{2\alpha}{(1-\alpha)}} \quad (\text{vi})$. Thus, the profit expression of each firm producing the intermediate good will be:

$$\pi_j = (p_j - 1)X_j \Leftrightarrow \pi_j = \left(\frac{1}{\alpha} - 1\right) H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{(1-\alpha)}}$$

$$\pi_j = H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\frac{1-\alpha}{\alpha} \alpha^{\frac{2}{(1-\alpha)}} \right] \text{ (vii)}$$

And $\pi_j = \pi$.

Given the Knowledge driven specification, labour is input in the R&D sector. Thus, the present value of the net benefits associated with the production of intermediate good j is:

$$V(t) = \int_s^\infty \exp[-r \times (t-s)] \pi_j(t) dt$$

$$= \int_s^\infty \exp[-r \times (t-s)] \pi dt, \text{ pois } \pi_j = \pi$$

$$= \frac{\pi}{r} \forall_j \text{ (viii)}$$

where π is the total profit of the final goods sector and r is the interest rate on each firm's assets.

Given equation vii and equation viii:

$$V(t) = \frac{H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\frac{1-\alpha}{\alpha} \alpha^{\frac{2}{(1-\alpha)}} \right]}{r(t)} \text{ (ix)}$$

As η is the deterministic number of resources that the inventor bears for a new prototype or a new idea measured against the final good Y , the resources allocated to R&D will be such that $V \geq \eta$. Thus, if: (i) $V > \eta$, there would be an infinite number of resources channelled to R&D; (ii) $V < \eta$, there would be no resources allocated to R&D and N would not change, N being the total number of innovations in each economy.

In equilibrium and for all t it turns out that $V(t) = \frac{\pi}{r(t)} = \eta$. This condition, called the free-entry condition means that η and $r(t)$ are constants and that $\bar{r}(s, t) = \frac{1}{t-s} \int_s^t r(w) dw$.

Thus, the no-arbitrage equation of the model will be $r(t) = \frac{\pi}{V(t)} + \frac{\dot{V}(t)}{V(t)}$ where $\frac{\pi}{V(t)}$ is the rate of profits and $\frac{\dot{V}(t)}{V(t)}$ are the diminishing capital losses from the change in value of the innovating firm. Since η is constant then $\dot{V}(t) = 0$, hence $r = r(t) = \frac{\pi}{\eta}$ and the expression

$$\text{of } r \text{ is such that } r = \frac{1}{\eta} \left[H_{jP}^{\frac{\beta}{1-\alpha}} H_{jS}^{\frac{\varepsilon}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\frac{1-\alpha}{\alpha} \alpha^{\frac{2}{(1-\alpha)}} \right] \right] \text{ (x)}.$$

Analysing now the consumer behaviour, given the interest rate on assets r , assets held by consumers b , income earned by them rb and labour income wH_f consumers have to decide between consuming and saving in order to maximise the utility $U = \int_0^\infty \left(\frac{c^{1-\theta}-1}{1-\theta} \right) \exp(-\rho t) dt$ (Afonso et al., 2019b). The Budget Constraint being $\dot{b} = rb + w_{fS}H_{fS} + w_{fP}H_{fP} - c$ and, solving the households' problem, we obtain the Euler

equation, where ρ is the growth rate of output Y and θ is the preference of consumers to consume in the present or in the future.

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \quad , \quad r > \rho$$

Turning to the R&D sector, a Knowledge Driven specification is considered, i.e., the input used in the production of knowledge is labour or human capital (e.g., Grossman & Helpman, 1989; Romer, 1990):

$$\dot{N} = \psi N^\phi H_{NP}^\mu H_{NS}^\vartheta$$

Where ψ corresponds to the labour productivity in R&D, H_{NP} corresponds to the workforce with private higher education employed in R&D and, H_{NS} corresponds to the workforce with private higher education employed in R&D. The parameters μ and ϑ reflect the duplication effect and are such that $\frac{\beta}{\varepsilon} = \frac{\mu}{\vartheta}$. The parameter ϕ translates the magnitude of spillovers, that is, the effect of the stock of accumulated knowledge on the production of new knowledge. In line with Jones (1995b) and semi-endogenous growth models, $\phi < 1$, implying that as the stock of knowledge increases, the production of new knowledge becomes increasingly difficult.

The growth rate of the number of intermediate goods depends on the existing technological knowledge, $N^{\phi-1}$ (proportional to the number of intermediate goods and the number of firms) and the number of researchers employed, in this case in the H_{NP}^μ and H_{NS}^ϑ .

$$\frac{\dot{N}}{N} = g_N = \psi N^{\phi-1} H_{NP}^\mu H_{NS}^\vartheta \quad (\mathbf{x1})$$

An innovation causes the government of each country to grant a patent, that is, the exclusive right to produce the new intermediate good, and any agent can bid for a patent and is willing to pay the present value of the profits obtained by the monopolist firm producing the intermediate good. Let p_N be the price of an innovation, determined by the arbitrage method, in equilibrium:

$$rp_N = \pi + \dot{p}_N \Leftrightarrow r = \frac{\pi}{p_N} + \frac{\dot{p}_N}{p_N}$$

p_N and π must grow at the same rate, in this case, at the rate of population growth n . Thus:

$$\begin{aligned} r = \frac{\pi}{p_N} + \frac{\dot{p}_N}{p_N} &\Leftrightarrow \pi = p(X)X - rX \Leftrightarrow \pi = \alpha \frac{Y}{NX_j} X_j - \left(\alpha^2 \frac{Y}{NX_j} \right) X_j \\ &\Leftrightarrow \pi = \alpha(1 - \alpha) \frac{Y}{N} \end{aligned}$$

On the other hand: $\frac{\dot{y}}{y} = \frac{\dot{N}}{N}$

Considering:

l_N : Share of researchers in R&D ($l_N = \frac{H_N}{H}$)

$(1-l_N)$: Share of employees in final goods ($1 - l_N = \frac{H_f}{H}$)

l_S : Share of employees with public education ($l_S = \frac{H_S}{H}$)

$(1-l_S)$: Share of employees with private education ($1 - l_S = \frac{H_P}{H}$)

l_N and l_S are constants in the steady-state and will be endogenously determined in the model.

$$\begin{aligned}
Y &= AX^\alpha H_{fP}^\beta H_{fS}^\varepsilon N^{1-\alpha} \\
\Leftrightarrow \frac{Y}{H} &= \frac{AX^\alpha H_{fP}^\beta H_{fS}^\varepsilon N^{1-\alpha}}{H} \\
\Leftrightarrow \frac{Y}{H} &= \frac{AX^\alpha N^{-\alpha} H_f^\beta H_{fP}^\beta H_{fS}^\varepsilon N}{H} \\
\Leftrightarrow \frac{Y}{H} &= \frac{AX^\alpha N^{-\alpha} H_f^\alpha H_{fP}^\beta H_{fS}^\varepsilon N}{H_f^\alpha H} \\
\Leftrightarrow \frac{Y}{H} &= A \left(\frac{X^\alpha}{N^\alpha H_f^\alpha} \right) H_f^\alpha H_{fP}^\beta H_{fS}^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha H_f^\alpha H_{fP}^\beta H_{fS}^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha ((1-l_N)H)^\alpha ((1-l_N)(1-l_S)H)^\beta ((1-l_N)l_S H)^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha ((1-l_N)H)^\alpha ((1-l_N)H)^{\beta+\varepsilon} (1-l_S)^\beta l_S^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha ((1-l_N)H)^{\alpha+1-\alpha} (1-l_S)^\beta l_S^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha (1-l_N)H(1-l_S)^\beta l_S^\varepsilon H^{-1} N \\
\Leftrightarrow y &= Ax^\alpha (1-l_N)(1-l_S)^\beta l_S^\varepsilon N \text{ (xii)}
\end{aligned}$$

Where $x = \frac{X}{NH}$.

Logarithmizing equation xii gives the following expression:

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{x}}{x} + \frac{(1-\dot{l}_N)}{(1-l_N)} + \beta \frac{(1-\dot{l}_N)}{(1-l_N)} + \varepsilon \frac{(\dot{l}_S)}{(l_S)} + \frac{\dot{N}}{N}$$

In steady state, the allocation of labour across sectors $(1-l_N)$, as well as the share from public education l_S , the level of A and the level of x are constant. Thus, the growth rate $\frac{\dot{Y}}{Y}$ is equal to the growth rate of N .

From the arbitrage condition we have that $\frac{\dot{\pi}}{\pi} = \frac{p_N}{p_N} \Leftrightarrow \pi = \alpha(1-\alpha) \frac{Y}{N} \Leftrightarrow \frac{\dot{\pi}}{\pi} = \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} - n$, so

$r = \frac{\pi}{p_N} + n \Leftrightarrow p_N = \frac{\pi}{r-n}$, which gives us the value of the patent along the balanced growth path.

The production function under study presents increasing returns considering both technological knowledge and ideas N , requiring that imperfect competition exists. In this case, firms are monopolists and sell intermediate goods at a price higher than marginal cost ($p = \frac{1}{\alpha}r > r$) and they do not consider the economy as a whole because the income generated compensates some input, i.e., wages, interest and income in the intermediate goods sector compensate for labour, capital, and innovations, respectively.

The marginal productivity of researchers in the R&D sector is given by $\bar{\psi}_S \equiv \psi \vartheta N^\phi H_{NP}^\mu H_{NS}^{\vartheta-1}$ and $\bar{\psi}_P \equiv \psi \mu N^\phi H_{NP}^{\mu-1} H_{NS}^\vartheta$. Note that the technological knowledge spillovers associated with ϕ are not internalised and, thus, the wage of a worker in the R&D sector is given by the product between the value of innovations p_N and the marginal product $\bar{\psi}_S$ and $\bar{\psi}_P$, respectively ($\bar{\psi}_S p_N = w_{NS}$ e $\bar{\psi}_P p_N = w_{NP}$). We will then have two distinct wages: one wage for workers with public education, $w_{fS} = w_{NS}$ and, one wage for workers with private education, $w_{fP} = w_{NP}$. For workers with the same type of education, the wage in the final goods sector must be the same as the wage in the R&D sector.

Thus, through wage equalization, we will determine endogenously, through equations ii and iii, the share of workers in the R&D sector, l_N considering that labour employed in the final goods sector earns a wage that is equal to its marginal product in that sector.

$$\begin{aligned}
w_{fS} = w_{NS} &\Leftrightarrow w_{fS} = \bar{\psi}_S p_N \\
&\Leftrightarrow \varepsilon \frac{Y}{H_{fS}} = \bar{\psi}_S \frac{\pi}{r-n} \\
&\Leftrightarrow \varepsilon \frac{Y}{H_{fS}} = \psi \vartheta N^\phi H_{NP}^\mu H_{NS}^{\vartheta-1} \alpha (1-\alpha) \frac{Y}{N(r-n)} \\
&\Leftrightarrow \frac{\varepsilon}{H_{fS}} = \frac{\psi \vartheta N^\phi H_{NP}^\mu H_{NS}^{\vartheta-1} \alpha (1-\alpha)}{(r-n)} \\
&\Leftrightarrow H_{fS} = \frac{\varepsilon(r-n)}{\psi \vartheta N^{\phi-1} H_{NP}^\mu H_{NS}^{\vartheta-1} \alpha (1-\alpha)} \quad \textbf{(xiii)}
\end{aligned}$$

$$\begin{aligned}
w_{fP} = w_{NP} &\Leftrightarrow w_{fP} = \bar{\psi}_P p_N \\
&\Leftrightarrow \beta \frac{Y}{H_{fP}} = \bar{\psi}_P \frac{\pi}{r-n} \\
&\Leftrightarrow \beta \frac{Y}{H_{fP}} = \psi \mu N^\phi H_{NP}^{\mu-1} H_{NS}^\vartheta \alpha (1-\alpha) \frac{Y}{N(r-n)} \\
&\Leftrightarrow \frac{\beta}{H_{fP}} = \frac{\psi \mu N^{\phi-1} H_{NP}^{\mu-1} H_{NS}^\vartheta \alpha (1-\alpha)}{(r-n)} \\
&\Leftrightarrow H_{fP} = \frac{\beta(r-n)}{\psi \mu N^{\phi-1} H_{NP}^{\mu-1} H_{NS}^\vartheta \alpha (1-\alpha)} \quad \textbf{(xiv)}
\end{aligned}$$

Through a system of equations xiii and xix, we have that:

$$\begin{aligned}
& \begin{cases} H_{fS} = \frac{\varepsilon(r-n)}{\psi \vartheta N \phi^{-1} H_{NP}^{\mu} H_{NS}^{\vartheta-1} \alpha (1-\alpha)} \\ H_{fP} = \frac{\beta(r-n)}{\psi \mu N \phi^{-1} H_{NP}^{\mu-1} H_{NS}^{\vartheta} \alpha (1-\alpha)} \end{cases} \\
& \Leftrightarrow \begin{cases} H_{fS} = \frac{\varepsilon(r-n)}{\vartheta g H_{NS}^{-1} \alpha (1-\alpha)} \\ H_{fP} = \frac{\beta(r-n)}{\mu g H_{NP}^{-1} \alpha (1-\alpha)} \end{cases} \\
& \Leftrightarrow \begin{cases} H_{fS} = \frac{\varepsilon(r-n) H_{NS}}{\vartheta g \alpha (1-\alpha)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} (1-l_N) l_S H = \frac{\varepsilon(r-n) l_N l_S H}{\vartheta g \alpha (1-\alpha)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} \frac{(1-l_N)}{l_N} = \frac{\varepsilon(r-n)}{\vartheta g \alpha (1-\alpha)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} (1-l_N) \vartheta g \alpha (1-\alpha) = l_N \varepsilon(r-n) \\ \end{cases} \\
& \Leftrightarrow \begin{cases} \vartheta g \alpha (1-\alpha) - \vartheta g \alpha (1-\alpha) l_N = l_N \varepsilon(r-n) \\ \end{cases} \\
& \Leftrightarrow \begin{cases} l_N = \frac{\vartheta g \alpha (1-\alpha)}{\varepsilon(r-n) + \vartheta g \alpha (1-\alpha)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} l_N = \frac{1}{1 + \frac{\varepsilon(r-n)}{\vartheta g \alpha (1-\alpha)}} \quad \text{(xv)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} (1-l_N)(1-l_S)H = \frac{\beta(r-n) l_N (1-l_S)H}{\mu g \alpha (1-\alpha)} \\ \end{cases} \\
& \Leftrightarrow \begin{cases} l_N = \frac{1}{1 + \frac{\beta(r-n)}{\mu g \alpha (1-\alpha)}} \quad \text{(xvi)} \\ \end{cases}
\end{aligned}$$

Equalising equations xv and xvi:

$$\begin{aligned}
& \frac{1}{1 + \frac{\varepsilon(r-n)}{\vartheta g \alpha (1-\alpha)}} = \frac{1}{1 + \frac{\beta(r-n)}{\mu g \alpha (1-\alpha)}} \\
& \Leftrightarrow \frac{\beta}{\varepsilon} = \frac{\mu}{\vartheta} \quad \text{(xvii)}, \text{ as we have already mentioned.}
\end{aligned}$$

Thus, we obtain the expression of l_N . Note that the model is correctly closed because, either the expression of l_N , or the expression of l_S do not depend on the variables H and

L . This fact must be verified because, otherwise l_N and l_S would not be constant in steady state since H and L grows to the growth rate of the population defined as n .

We know that in steady state $\frac{\dot{y}}{y} = \hat{y} = g = \hat{N} = \frac{\dot{N}}{N}$ and that $\frac{\dot{N}}{N} = g_N = \psi N^{\phi-1} H_{NP}^{\mu} H_{NS}^{\vartheta}$. In steady state, the growth rate of $\frac{\dot{N}}{N}$ must be constant. Thus:

$$\begin{aligned}
g_{\frac{\dot{N}}{N}} &= 0 \\
\Leftrightarrow (\phi - 1) \frac{\dot{N}}{N} + \mu \frac{\dot{H}_{NP}}{H_{NP}} + \vartheta \frac{\dot{H}_{NS}}{H_{NS}} &= 0 \\
\Leftrightarrow (\phi - 1)g + \mu n + \vartheta n &= 0 \\
\Leftrightarrow (\phi - 1)g + n(\mu + \vartheta) &= 0 \\
\Leftrightarrow g &= \frac{(\mu + \vartheta)}{(1 - \phi)} n
\end{aligned}$$

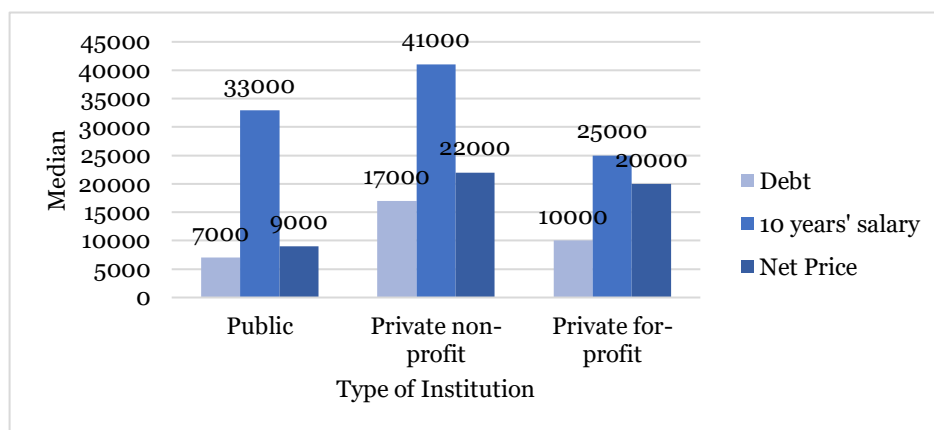
The economic growth rate is then given by the expression $g = \frac{(\mu + \vartheta)}{(1 - \phi)} n$ **(xviii)**.

4. Public versus Private higher education: Possible scenarios for analysis

There are indeed differences between private and public universities that sometimes have similar characteristics, but also different attributes. Tang (2012) states that private higher education has evolved faster than public higher education and can be considered as a complement to the latter.

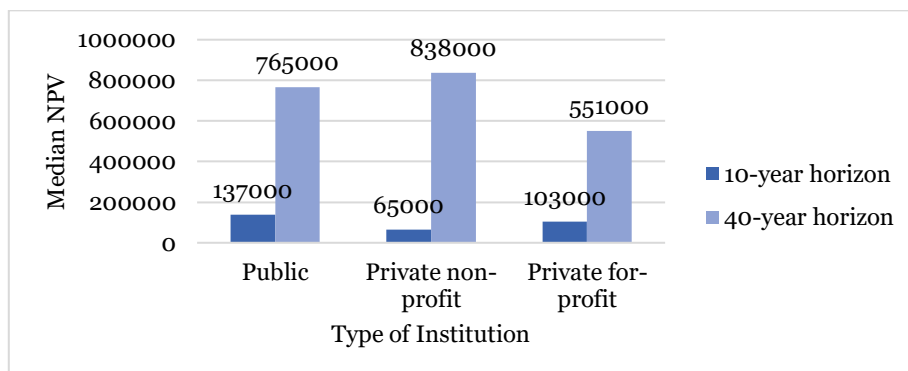
Given the reality of the United States of America, for example, Private Universities have an advantage over public ones: individuals graduating from private non-profit universities have a considerable advantage in earnings 10 years after enrolment with average earnings of \$41,000, while for the same period, for an individual from a public university the earnings are \$33,000 (see graph 1) (Carnevale et al., 2019). However, although debt levels at public higher education institutions are less than half that at private ones (\$7,000 versus \$17,000, respectively), the long-term gains of students attending private institutions are sufficient to offset their costs (see Figure 2).

Figure 1 - Costs and Debt levels for students, considering the type of institution, in USA



Source: Carnevale et al. (2019)

Figure 2 - The net present value of future earnings for students, in \$, considering the type of institution, in USA



Source: Carnevale et al. (2019)

According to the report "A First Try at ROI: Ranking 4,500 Colleges" by the Georgetown Center on Education and the Workforce by Carnevale et al. (2019), there are considerable differences in the value of degrees earned at different types of universities, such as for-profit, private non-profit and public non-profit universities. According to the same report, the economic gains for a student at a private for-profit university are around \$551000 compared to \$765000 at a public university and \$838000 at a private non-profit university (Carnevale et al., 2019). The report also mentions a higher return on investment in higher education in the long term (10-year horizon) at private non-profit universities compared to public universities even though the tuition fee at the latter is considerably lower (Carnevale et al., 2019).

Table 1 - Top 20 universities in the USA, considering potential early and mid-career salary of alumni

Ranking	Higher Education Institution	Public vs. Private	Remuneration at the beginning of the career (in \$)	Remuneration at mid-career (in \$)
1	<i>Stanford University</i>	Private	86800	164200
2	<i>Massachusetts Institute of Technology</i>	Private	92300	163900
3	<i>United States Naval Academy</i>	Public	85800	160700
4	<i>Harvey Mudd College</i>	Public	93100	160600
5	<i>California Institute of Technology</i>	Private	93200	159900
6	<i>Harvard University</i>	Private	80100	159400
7	<i>United States Military Academy</i>	Public	84800	158100
8	<i>Princeton University</i>	Private	79900	157200
9	<i>SUNY Maritime College</i>	Public	76100	156500
10	<i>Yale University</i>	Private	77100	154600
11	<i>Babson College</i>	Private	73900	153200
12	<i>University of Pennsylvania</i>	Private	76800	152900
13	<i>Dartmouth College</i>	Private	75500	152500
14	<i>Claremont McKenna College</i>	Private	73800	150000
15	<i>Georgetown University</i>	Private	69400	148900
16	<i>Santa Clara University</i>	Private	72900	148600
17	<i>Stevens Institute of Technology</i>	Private	77800	147700
18	<i>University of California-Berkeley</i>	Private	74500	146700
19	<i>United States Air Force Academy</i>	Public	81100	146200
20	<i>Colgate University</i>	Public	70000	145400

Source: College Salary Report (2020)

According to the report "College Salary 2020" (College Salary Report, 2020), which analyses the potential salary at the beginning and middle of the career of alumni of the best USA universities of any type and which is based on the PayScale Salary server, let us focus our attention on the first 20 on the same list. Of the sample of 20 universities, 14 of them are private and it is these that have the highest potential mid-career salary. On the other hand, the public universities mentioned in Table 1 show higher potential early career pay for alumni (College Salary Report, 2020).

Looking at the QS World University Ranking 2021 (Qs Top Universities, 2021), which considers the best universities in the world, both public and private, let us focus our attention on their reputation for employers and the type of university, public or private (Table 2). In terms of employer reputation, private universities also have a higher ranking than public ones, and the top 5 best universities in the world are made up of 5 private universities.

Table 2 - Employer reputation of the top 15 universities in the world, according to the QS World University 2021 Ranking

Ranking	Higher Education Institution	Country	Public vs. Private	Employer Reputation
1	<i>Massachusetts Institute of Technology (MIT)</i>	USA	Private	100
2	<i>Stanford University</i>	USA	Private	100
3	<i>Harvard University</i>	USA	Private	100
4	<i>California Institute of Technology (Caltech)</i>	USA	Private	82,8
5	<i>University of Oxford</i>	United Kingdom	Public	100
6	<i>ETH Zurich - Swiss Federal Institute of Technology</i>	Switzerland	Public	96,6
7	<i>University of Cambridge</i>	United Kingdom	Public	100
8	<i>Imperial College London</i>	United Kingdom	Public	99,8
9	<i>University of Chicago</i>	USA	Private	91,3
10	<i>UCL</i>	United Kingdom	Public	98,3
11	<i>National University of Singapore (NUS)</i>	Singapore	Public	98,4
12	<i>Princeton University</i>	USA	Private	99
13	<i>Nanyang Technological University, Singapore (NTU)</i>	Singapore	Public	89,8
14	<i>EPFL</i>	USA	Public	80
15	<i>Tsinghua University</i>	China	Public	98,6

Source: Qs Top Universities (2021)

Considering the existing data mentioned above and according to the verified world reality, it becomes relatively important to differentiate three possible scenarios for analysis: the European Union (EU) scenario, the Southeast Asia scenario, and the United States of America (USA) scenario. Let us now briefly analyse each of these.

In the context of the EU, the European Commission has boosted mobility and exchanges between students from the 27 member states of the EU (European Commission, 2021), through the European Education Area. To this end, it has a network of European universities, public and private, which aim to establish a relationship between universities and their students' own future, personal and professional (European Commission, 2021).

Most national policies in Europe encourage countries to turn to private sources of funding. However, public funding still represents a significant share of higher education budgets (Eurodyce, 2008). According to data available for the 27 EU Member States in

2003, 79,9% of funding for higher education institutions came from public sources (Eurodyce, 2008). However, the influence that this funding has on institutions is not the same for all of them (Eurodyce, 2008). In the last 20 years, many EU countries have developed mechanisms and formulas to calculate the amounts efficiently allocated to higher education, as well as measures that relate the level of public funding and the performance of the various institutions (Eurodyce, 2008). In this scenario, the relationship between the parameters ε , β , ϑ and μ is not very clear. Note that ε represents the income share of publicly trained final goods workers in the total income of a country and β the share of privately trained final goods workers in the total income of a country while μ reflects the productivity of privately trained researchers in knowledge production and ϑ the productivity of publicly trained researchers in knowledge production. Thus, the parameters for the EU scenario take the values such that $\varepsilon > \beta$ and $\vartheta > \mu$.

The Southeast Asia Region, consisting of the innovative countries Japan, South Korea, Singapore, and Malaysia, moves in a similar direction as the EU. It is indeed remarkable that public higher education institutions compete with private ones, both being in a process of simultaneous growth (Naidu & Deran, 2016). Take Malaysia for example where 60% of undergraduates come from a public university, while the remaining 40% from a private one, denoting a closeness in the two types of institutions mentioned (Da, 2007). Note that Singapore, for example, has two public universities among the 15 bests in the world (see Table 3), which are indicated as two of the best universities in Asia and where the participation of private funding is very low (Cerqueira, 2021; International higher education, 2021). Thus, and like the EU scenario, the parameters have the following behaviour: $\varepsilon > \beta$ and $\vartheta > \mu$

Although with a rudimentary empirical basis, but to sustain the private' logic, the argument usually used points out that in the United States the main universities are private and some are considered among the most prestigious in the world (Schneider et al., 2018). The character of most of these institutions is non-profit and they rely, to this day, on the collaboration of public funding in the areas of research and innovation (Schneider et al., 2018). Thus, we consider for the North American scenario that the parameters take values such that $\varepsilon < \beta$ and $\vartheta < \mu$.

5. Results and Discussion

Recurring to a calibration method and using equations xvii and xviii and the condition $\beta + \varepsilon = (1 - \alpha)$, the following sums were calculated: the sum of the productivities associated with workers in the R&D sector with education in private and public higher education, respectively, $(\mu + \vartheta)$, the sum of the shares associated with workers in the final good sector with education in private and public higher education, respectively, $(\beta + \varepsilon)$, through the value found by Neves and Sequeira (2018) for spillovers, ϕ and, the initial calculation of the economic growth rate, g , and the population growth rate, n . For the respective calculation, we used the data presented in Table 3, in the 1960-2019 period for the previously mentioned scenarios.

Table 3 - Variables description

Variable	Scenario	Description	Source
pop	Southeast asia, USA	Population (in millions)	Penn World Table (2021)
POP	EU	Population, total	World Bank (2021)
rgdpna	Southeast asia, USA	Real GDP at constant national prices (in mil. 2017US\$)	Penn World Table (2021)
gdpgr	EU	GDP per capita growth (annual %)	World Bank (2021)
Labsh	Southeast asia, USA	Share of labour compensation in GDP at current national prices (equivalent to $1 - \alpha$)	Penn World Table (2021)
shareL	EU	Labor force participation rate, total (% of total population ages 15+) (national estimate)	World Bank (2021)
ϕ	all	Spillovers in the production of knowledge	Neves e Sequeira (2018)

To analyse each of the scenarios using the calibration method, a sample of 3 different areas was chosen for each one to understand the behaviour of the parameters under study. The sample selected for each scenario is shown in Table 4. Through the example 1 mentioned below, we calculated the value of the parameters mentioned in Table 4, starting from a given value of ε considering the evidence mentioned in section 4.

Example 1 for the USA scenario: Starting from the value of $\varepsilon = 0,2$ which means that the share of the income of publicly educated final goods workers in the total income of the country is 0,2 and, considering the following values already calculated through the equations found throughout the model: $\mu + \vartheta = 0,360619$ and $\beta + \varepsilon = 0,61639$. Thus, $\beta = 0,61639 - 0,2 = 0,41639$. We know that $\frac{\varepsilon}{\vartheta} = \frac{\beta}{\mu} \Leftrightarrow \frac{0,2}{0,41639} = \frac{\vartheta}{\mu} \Leftrightarrow \frac{\vartheta}{\mu} = 0,48032 \Leftrightarrow \vartheta = 0,48032\mu$ and thus: $\mu + \vartheta = 0,360619 \Leftrightarrow \mu + 0,48032\mu = 0,360619 \Leftrightarrow \mu = 0,243609$

and $\vartheta = 0,48032 * 0,243609 = 0,11701$. Thus: $\varepsilon = 0,2$, $\beta = 0,41639$, $\mu = 0,243609$ and $\vartheta = 0,11701$.

Table 4 – Scenarios for analysis, for the period 1960-2019, using the calibration method

Scenario	Parameter	Value
EU	$\beta + \varepsilon$	0,561580979
	$\mu + \vartheta$	1,191028384
	ε	0,35
	β	0,211580979
	μ	0,4487313
	ϑ	0,742297084
Southeast asia	$\beta + \varepsilon$	0,47850555675
	$\mu + \vartheta$	0,02237532225
	ε	0,3
	β	0,17850555675
	μ	0,00834707
	ϑ	0,014028252
United States of America	$\beta + \varepsilon$	0,61639
	$\mu + \vartheta$	0,360619
	ε	0,2
	β	0,41639
	μ	0,243609
	ϑ	0,11701

Note that the values presented in Table 4 for the sums $(\beta + \varepsilon)$ and $(\mu + \vartheta)$ correspondent to the different scenarios, including the Southeast Asia scenario and the 4 countries that constitute it, are the average values calculated from the values in Table 5 present in the Appendices section.

For a time horizon of 1960-2019 and, for the three different scenarios, we intended to analyse the influence of a given value of the share of the income of final good workers with public training at higher education level in the total income of a country, ε , on the behaviour of the other parameters such as β , which represents the share of the income of final good workers with private training in the total income of a country, μ , which reflects the productivity of researchers with private training in knowledge production and, finally, the productivity of researchers with public training in knowledge production, ϑ .

In the calibration method, the values used for ε are in line with the facts and realities described in section 4. Note that the value mentioned for ε of the EU and Southeast Asia

scenarios is very close to the value of β given the importance attributed to each type of higher education, both public and private, is also very similar.

The behaviour of the parameters μ and ϑ , which reflect the productivity of each type of researcher in knowledge production, private and public, respectively, directly influences the economic performance of the countries, given by their GDP per capita growth rate. According to equation xviii of the model under analysis, a country, or in this case, an aggregate of countries, has a higher economic growth rate the greater the sum of the parameters $\mu + \vartheta$, for a given value of n , i.e., the value of the economic growth rate varies in the same direction as the sum of the productivity of workers in the R&D sector, whatever their level of education.

Let us look at the sum of $\mu + \vartheta$, shown in Table 4, for each of the scenarios, as well as the values of the average annual growth rate of GDP per capita, g , and the average population growth rate, n , shown in Table 5 (Appendices section). The EU has an average annual population growth rate of 0.385% and an average annual GDP per capita or economic growth rate of 1.88%. In this scenario, μ and ϑ have higher values than β and ε , suggesting that in the EU, the productivity of researchers, belonging to the R&D sector, is significantly higher than that of workers in the final goods sector. These values indicate that the EU relies on workers with a higher level of qualification belonging to the R&D sector and, on the other hand, the income share of workers in the same sector with public training overlaps, albeit by a minimal difference, with that of workers who, in the same sector, attended a private higher education institution. This can be explained by the fact that most of the universities in the EU are public. On the other hand, the minimal difference observed may be due to the European Region Action Scheme for the Mobility of Universities (ERASMUS), which links public and private higher education institutions across the EU.

We will now analyse the US scenario. In this scenario, the ε parameter exhibits a behaviour contrary to that of the scenarios analysed above, given the configuration mentioned in section 4 which highlights the higher importance given to private higher education institutions compared to public ones. On analysing Table 4 in general terms regarding the parameters calculated, we observe that, both in the final goods sector and in the R&D sector, the parameters corresponding to workers with private training are approximately twice as high as those corresponding to workers with public training. Note that such findings are in line with the data in Table 1 as the best US universities are private and it is in these universities that mid-career pay is highest, i.e., they have both a β and a μ higher than ε and ϑ , respectively. In addition, μ and ϑ are higher than those of Southeast Asian countries, but lower than those of the EU.

Last but not least, let us analyse the Southeast Asia scenario which, as previously mentioned, presents a similar behaviour to the EU scenario, i.e., the value mentioned for the share of the income of final good workers with public education at tertiary level in the total income of a country, ε , indicated in table 4, is higher but very close to the value of the share of the income of final good workers with private education at tertiary level in the total income of a country, β . This choice is due to the fact that the aggregate of countries under analysis has education programmes at higher education level similar to those existing in the EU, such as ERASMUS. It should be noted that the value found for the β parameter, approximately 22,86%, does not differ much from the value of the ε parameter, which is 25%, meaning that the income shares of workers in the final goods sector, both those with public and private higher education, contribute in a similar way to total income in Southeast Asia. As regards the values of the parameters that concern the R&D sector, their contribution to economic growth in the region in question is not so perceptible because the values are quite small, that is, the productivity levels of each type of worker present a value of approximately 1% for the scenario under analysis. It should be noted that in the Southeast Asian scenario, the parameters μ and ϑ present low values, lower than those of the EU, which indicates low levels of productivity of researchers, both with public and private training.

Focusing our attention on expression xvii and Table 4, we observe that the scenario where the influence of the productivity of researchers on steady state economic growth is greatest is the EU scenario, a result that may derive from the constant policies to promote the R&D sector and the continuous growth of research areas.

Note that, for the period 1960-2019 (Table 4), the productivity of Asian researchers, both those with public and private training, is lower than that of EU and USA researchers. This may be due to the fact that, for the period analysed, these countries, some of them known as Asian tigers, belonging to the Association of Southeast Asian Nations (ASEAN) have low-cost labour due to its abundance, which suggests that high productivity of researchers is not necessary (ASEAN, 2021). On the other hand, this last statement is further reinforced with the high values found for the parameters ε and β , which represent the shares of the income of workers in the final goods sector in the total income of the country with public and private training, respectively. In the Asian countries scenario, we observe higher values in the final goods sector than in the R&D sector, which may be related to the specification chosen for the model, horizontal innovation. In the case of Southeast Asia, horizontal innovation, that is, the creation of a new product for the specific analysis, leads to the values mentioned in Table 4 for parameters μ and ϑ . If the Knowledge driven specification with vertical innovation was chosen, the values of parameters μ and ϑ , related to the R&D sector, specifically the productivity of researchers

with private and public training, respectively, could be higher because, in this scenario we observe a greater evolution/innovation at the technological level of final goods, since this scenario includes two of the most innovative countries in the world, South Korea and Singapore, according to the Bloomberg Global Innovation Index, which assesses issues such as patent registration, investment in education and productivity in the most innovative countries in the world (Iberdrola, 2021). This data is also in line with the Entrepreneurship sub-rank in the 2021 Best Countries report which indicates that Japan, South Korea and China are the countries with the highest technological expertise worldwide (US News, 2021).

Comparing, for the three scenarios under analysis, the value of g , mentioned in table 5, with the parameters μ and ϑ regarding the R&D sector, mentioned in table 4, we observe that a scenario with a lower economic growth rate presents a higher sum of researchers' productivity ($\mu + \vartheta$). This statement may be related to the size of the scenarios under analysis, represented in the model by the population growth rate, n , and to the number of higher education institutions, both public and private, in each scenario.

6. Conclusion

In this study we built on an endogenous economic growth model, with horizontal Knowledge Driven R&D and, to incorporate the differentiation, public versus private, imposed on the tertiary education level of workers both in the R&D sector and in the final goods sector. One of the main conclusions of the model is that, for a given economy in steady state, the ratio between the income share of workers in the final goods sector with public training in the country's total income and the income share of workers in the final goods sector with private training equals the ratio between the productivity of scientists with public training and the productivity of scientists with private training. Given the value assigned to the ε parameter and the facts collected for each of the scenarios, the results obtained support the various equations found throughout the model. The particularity of the choice of scenarios under analysis allowed the behaviour of the various parameters to be analysed in quite different higher education contexts, leading to the conclusion that countries where higher education training is mostly private, as is the case of the USA, have lower productivity levels for scientists than workers from countries where private higher education is small, as is the case of the EU, possibly due to the fact of the inclusive mobility policies among students from the various member states of the latter. Thus, countries and their policy makers should increasingly seek to produce policies and measures to promote education at higher education level that are more inclusive of both public and private institutions.

This study provides an opportunity for further research in endogenous economic growth theoretical models because, through this model, policy makers can create better public funding conditions for each type of higher education institutions to positively influence both economic growth and level of innovation of their country. Furthermore, the modelling of the model would be essential so as not to have to resort to a calibration method for further analysis of the various parameters for the different scenarios.

Appendices

Table 5 - Results of each scenario, using equation xviii and the variables present in table 3, for the period 1960-2019

years	United States of America			European Union			Japan			Malaysia			Singapore			South Korea			South East Asia									
	n	g	μ+θ	n	g	μ+θ	n	g	μ+θ	n	g	μ+θ	n	g	μ+θ	n	g	μ+θ	n	g	μ+θ							
	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε	β+ε							
1960			0.636742						0.620528			0.305726			0.402062			0.64677										
1961	0.016715	0.008681	0.089324	0.633703	0.008664		0.009051	0.108498	2.061761	0.620528	0.027871	0.043747	0.269977	0.334014	0.044306	0.220406	0.402062	0.033477	0.011245	0.057772	0.64677	0.026103	0.051949	0.653389	0.493772			
1962	0.015499	0.044969	0.499049	0.629107	0.008895		0.009358	0.075899	1.395039	0.620528	0.028047	0.03239	0.198634	0.305726	0.028078	0.040461	0.247855	0.402062	0.031486	-0.008	-0.04369	0.64677	0.024242	0.035188	0.449458	0.493772		
1963	0.014496	0.028629	0.3397	0.626385	0.00913		0.010228	0.076859	1.292487	0.620528	0.027179	0.038084	0.241014	0.305726	0.025597	0.071937	0.483384	0.402062	0.029939	0.046585	0.267627	0.64677	0.023236	0.058366	0.571128	0.493772		
1964	0.013987	0.043081	0.529753	0.624379	0.009096		0.010477	0.100328	1.647045	0.620528	0.027219	0.022134	0.139868	0.305726	0.025961	-0.06276	-0.41582	0.402062	0.028396	0.049235	0.298231	0.64677	0.023013	0.027234	0.41733	0.493772		
1965	0.01258	0.051766	0.707763	0.61936	0.008605		0.010797	0.045442	0.723885	0.620528	0.026623	0.045926	0.296708	0.305726	0.024598	0.049041	0.342914	0.402062	0.025765	0.023699	0.15821	0.64677	0.021946	0.041027	0.380429	0.493772		
1966	0.011616	0.053694	0.795068	0.622895	0.008059		0.00918	0.092455	1.732209	0.620528	0.026136	0.044303	0.291558	0.305726	0.025174	0.08047	0.549818	0.402062	0.025466	0.086421	0.583696	0.64677	0.021489	0.075912	0.78932	0.493772		
1967	0.010948	0.01631	0.256225	0.630079	0.007656		0.010371	0.099436	1.649048	0.620528	0.025766	0.008679	0.057939	0.305726	0.022333	0.09772	0.752618	0.402062	0.023611	0.031352	0.228395	0.64677	0.02052	0.059297	0.672	0.493772		
1968	0.010035	0.038668	0.662801	0.634226	0.007092		0.011261	0.106628	1.628628	0.620528	0.025058	0.046645	0.320174	0.305726	0.017395	0.116176	1.148746	0.402062	0.023464	0.080867	0.59278	0.64677	0.019295	0.087579	0.922582	0.493772		
1969	0.009082	0.021377	0.37441	0.643875	0.006844		0.011879	0.106389	1.540483	0.620528	0.024302	0.025491	0.180419	0.305726	0.015159	0.118826	1.348242	0.402062	0.022894	0.117684	0.884155	0.64677	0.018558	0.092098	0.988325	0.493772		
1970	0.011718	-0.00959	-0.14076	0.648986	0.005481		0.011371	0.090425	1.36781	0.620528	0.023251	0.034109	0.252319	0.305726	0.01567	0.119592	1.312643	0.402062	0.022096	0.060689	0.472415	0.64677	0.018097	0.076204	0.851296	0.493772		
1971	0.008929	0.023792	0.458319	0.637649	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356	0.031085	0.008929	0.005356		
1972	0.008921	0.043282	0.834499	0.639445	0.006702	0.040886	1.049289		0.014554	0.068584	0.810532	0.620528	0.02367	0.068591	0.498431	0.305726	0.018731	1.11232	0.031382	0.402062	0.019963	0.051159	0.440782	0.633555	0.019229	0.075164	0.695282	0.490468
1973	0.008951	0.047085	0.90476	0.640667	0.00647	0.063519	1.427757		0.014378	0.065013	0.777731	0.620528	0.023698	0.091153	0.661592	0.305726	0.018183	0.086274	0.61895	0.402062	0.019327	0.127198	1.132027	0.618852	0.018896	0.09241	0.846861	0.486792
1974	0.008998	-0.01427	-0.27288	0.640969	0.006161	0.02466	0.688406		0.013642	-0.02535	0.32209	0.620528	0.024021	0.057778	0.413713	0.305726	0.016793	0.04365	0.447079	0.402062	0.018352	0.075383	0.70651	0.600907	0.018202	0.037816	0.311304	0.483206
1975	0.009057	-0.01101	-0.20913	0.625561	0.005979	-0.01237	-0.35588		0.012526	0.018162	0.248394	0.620528	0.02452	-0.01611	-0.11304	0.305726	0.014977	0.024474	0.281065	0.402062	0.017187	0.060178	0.602237	0.61161	0.017303	0.021675	0.254915	0.484982
1976	0.009153	0.044323	0.832933	0.621559	0.005415	0.043334	1.376426		0.011261	0.028127	0.430318	0.620528	0.025201	0.088211	0.602044	0.305726	0.012896	0.060693	0.809506	0.402062	0.015924	0.114468	1.236405	0.61196	0.01632	0.072886	0.769568	0.485069
1977	0.009269	0.036633	0.679788	0.621596	0.004874	0.032783	0.839239		0.010104	0.033461	0.569594	0.620528	0.025794	0.050435	0.336307	0.305726	0.011414	0.056468	0.850897	0.402062	0.01484	0.106933	1.239372	0.615975	0.015538	0.061824	0.749043	0.486073
1978	0.009356	0.045571	0.837776	0.622295	0.004588	0.025871	0.969919		0.009156	0.043168	0.81092	0.620528	0.025998	0.039513	0.261413	0.305726	0.011411	0.056513	0.988969	0.402062	0.014168	0.09403	1.141547	0.620217	0.015183	0.060581	0.08071	0.487133
1979	0.009395	0.022059	0.423839	0.62262	0.004403	0.033898	1.324116		0.008527	0.045922	0.926253	0.620528	0.025695	0.060697	0.44245	0.305726	0.013321	0.08114	1.04767	0.402062	0.014033	0.071673	0.878456	0.619259	0.015394	0.066208	0.823707	0.486894
1980	-0.0094	-0.01186	-0.21696	0.623439	0.004492	0.016144	0.618165		0.008125	0.019889	0.421018	0.620528	0.025115	0.048118	0.295933	0.305726	0.016477	0.083283	0.886937	0.402062	0.014243	-0.03027	-0.36552	0.585542	0.01599	0.030255	0.031360	0.478465
1981	0.009411	0.015817	0.289064	0.614216	0.004109	0.009008	0.037993		0.007774	0.034055	0.753467	0.61841	0.023459	0.043999	0.310803	0.305726	0.02024	0.086177	0.732346	0.41415	0.014632	0.056996	0.669992	0.588707	0.016749	0.055307	0.616652	0.481748
1982	0.009436	-0.02721	-0.49594	0.616743	0.003281	0.004862	0.037993		0.007348	0.025589	0.599004	0.622449	0.023845	0.035999	0.052768	0.305726	0.023365	0.04657	0.34285	0.451699	0.014842	0.067537	0.782681	0.584444	0.017352	0.034688	0.494326	0.49108
1983	0.009445	0.036054	0.656584	0.603852	0.002626	0.011751	0.769778	0.546284	0.006876	0.028161	0.704452	0.62604	0.024062	0.037208	0.065963	0.305726	0.024991	0.059079	0.406607	0.465938	0.014616	0.11743	1.38191	0.584819	0.017636	0.060469	0.689733	0.495631
1984	0.009434	0.062345	1.136713	0.601952	0.002323	0.022477	1.664439	0.551746	0.006324	0.038452	1.045762	0.614821	0.025161	0.051171	0.349798	0.305726	0.024637	0.061765	0.431214	0.477885	0.013798	0.09047	1.127776	0.572506	0.01748	0.060465	0.738638	0.492735
1985	0.009414	0.031981	0.584313	0.602302	0.002405	0.02105	1.505362	0.546922	0.005728	0.04634	1.391408	0.605987	0.026765	-0.03654	-0.23483	0.305726	0.023081	-0.02865	-0.21349	0.491038	0.012611	0.064958	0.885964	0.565455	0.017046	0.011527	0.457264	0.492052
1986	0.009411	0.02498	0.456559	0.607737	0.002783	0.023015	1.425227	0.551716	0.005133	0.027989	0.937849	0.596799	0.028515	-0.017	-0.10255	0.305726	0.021032	-0.00745	-0.0609	0.448351	0.011291	0.100843	1.53622	0.54774	0.016493	0.026096	0.577655	0.474654
1987	0.009432	0.024929	0.454619	0.615973	0.002952	0.022255	1.296522	0.552626	0.0046	0.042511	1.589385	0.588817	0.02981	0.023397	0.134996	0.305726	0.019741	0.086529	0.753914	0.424182	0.010213	0.115841	1.950877	0.542283	0.016091	0.067069	1.120293	0.465252
1988	0.009465	0.032003	0.581545	0.620666	0.003246	0.038315	2.030448	0.556323	0.004159	0.063428	2.62344	0.579881	0.030291	0.055998	0.317797	0.305726	0.021999	0.090832	0.781566	0.415926	0.009598	0.109231	1.957488	0.554932	0.016009	0.079872	1.402092	0.464116
1989	0.009514	0.026956	0.487316	0.611859	0.0034	0.03572	1.80691	0.55639	0.003835	0.044574	1.98995	0.577071	0.029755	0.060255	0.348314	0.305726	0.022272	0.077585	0.599157	0.425719	0.009612	0.060535	1.083238	0.568674	0.016369	0.060738	1.007415	0.469298
1990	0.009589	0.009183	0.164725	0.615198	0.003352	0.030249	1.552048	0.550933	0.003601	0.045164	2.157297	0.573507	0.028573	0.059814	0.360054	0.305726	0.025769	0.070622	0.471384	0.429348	0.010054	0.087838	1.502647	0.569839	0.016999	0.065859	1.122845	0.469605
1991	0.009595	-0.01058	-0.18959	0.615099	0.002979	0.01506	0.869561	0.55749	0.00341	0.030361	1.546626	0.583021	0.027183	0.066464	0.420546	0.305726	0.029267	0.036548	0.21479	0.438174	0.010595	0.096166	1.561134	0.569507	0.017614	0.05746	0.935774	0.474107
1992	0.00963	0.02355	0.452771	0.620048	0.002925	0.00851	0.500486	0.567871	0.003214	0.002525	1.280498	0.591112	0.026065	0.061191	0.403798	0.305726	0.023176	0.03357	0.181796	0.447074	0.010953	0.050861	0.792717	0.568699	0.017998	0.037623	0.414815	0.478153

References

- Acemoglu, D. & Robinson, J. A. (2012). *Why Nations Fail: Origins of Power, Poverty and Prosperity*. Crown Publishers, New York.
- Afonso, O., Gil, P. M., Neves, P. C. & Sequeira, T. N. (2019a). *Demographic Change, Wage Inequality, and Technology*. In A. Bucci et al. (eds.), *Human Capital and Economic Growth*, Cap. 4, p. 89-135.
- Afonso, O., Vasconcelos, P. B., Neves, P. C. & Sequeira, T. N. (2019b). *Crescimento Económico: Uma abordagem moderna e novas tendências*. Coimbra: Edições Almedina.
- Aghion, P. & Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60, pp. 323-351.
- Alesina, A & Rodrik, D. (1992). Distribution, political conflict and economic growth: a simple theory and some empirical evidence. In: Cukierman, A., Hercowitz, Z., Leiderman, L. (ed): *Political Economy, Growth, and Business Cycles*, The MIT Press (Part I), Cambridge.
- Arcalean, C. & Schiopu, I. (2016). Inequality, opting-out and public education funding. *Social Choice and Welfare*, 46 (4), pp. 811–837.
- ASEAN (2021). Association of Southeast Asian Nations – overview. Available at: <https://asean.org/asean/about-asean/overview/>.
- Batabyal, A. A. & Nijkamp, P. (2013). Human capital use, innovation, patent protection, and economic growth in multiple regions. *Economics of Innovation and New Technology*, 22(2), pp. 113–26.
- Baten, J. & Hippe, R. (2018). Geography, land inequality and regional numeracy in Europe in historical perspective. *Journal of Economic Growth*, 23(1), pp. 79–109.
- Blankenau, W. F. S. & Nicole, B. (2004). Public education expenditures and growth. *Journal of Development Economics*, 73 (2), pp. 583–605.
- Bond-Smith, S. (2019). The decades-long dispute over scale effects in the theory of economic growth. *Journal of Economic Surveys*, 0(0), pp. 1-30.
- Bronzini, R. & Piselli, P. (2009). Determinants of long-run regional productivity with geographical spillovers: the role of R&D, human capital and public infrastructure. *Regional Science and Urban Economics*, 39 (2), pp. 187–199.
- Carnevale, A. P., Cheah, B. & Van Der Werf, M. (2019). *A first try at ROI: Ranking 4,500 Colleges*. Center on Education and the Workforce, Georgetown University. Available at: https://1gyhoq479ufd3yna29x7ubjn-wpengine.netdna-ssl.com/wp-content/uploads/College_ROI.pdf.
- Cerqueira, R. (2021). Conheça as melhores universidades do Sudeste Asiático. Universidade do Intercâmbio. Available at: <https://www.universidadedointercambio.com/sudeste-asiatico-uni/>.

Chiu, J., Meh, C. & Wright, R. (2017). Innovation and growth with financial, and other, frictions. *International Economic Review*, 58 (1).

College Salary Report (2020). *Best Universities and Colleges by Salary Potential*. Available at: <https://www.payscale.com/college-salary-report/all-bachelors>.

Cozzi, G. & Spinesi, L. (2006). Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics*, 10(1), pp. 39-55.

Da, W. (2007). Public and private higher education institutions in Malaysia: Competing complementary or crossbreeds as education providers. *Kajian Malaysia*, xxv.

Das, M. (2007). Persistent inequality: an explanation based on limited parental altruism. *Journal of Development Economics*, 84 (1), pp. 251-270.

de la Croix, D. & Doepke, M. (2004). Public versus private education when differential fertility matters. *Journal of Development Economics*, 73 (2), pp. 607-629.

de la Croix, D. & Doepke, M. (2009). To segregate or to integrate: education politics and democracy. *Review of Economic Studies*, 76 (2), pp. 597-628.

Denti, D. (2010). R&D spillovers and regional growth. In: Capello R, Nijkamp P, editors. *Handbook of regional growth and development theories* (pp. 211-36). Cheltenham, England: Edward Elgar Publishing.

Dinopoulos, E. & Thompson, P. (1998). Schumpeterian growth without scale effects. *Journal of Economic Growth*, 3(4), pp. 306-334.

Dissou, Y., Didic, S. & Yakautsava, T. (2016). Government spending on education, human capital accumulation, and growth. *Economic Modelling*, 58, pp. 9-21.

Domar, E. D. (1946). Capital Expansion, Rate of Growth and Employment. *Econometrica*, 14 (2), pp. 137-147.

Drucker, P. F. (1966). The Effective Executive. *Harper and Row*, pp.178.

Epple, D. & Romano, R. E. (1996a). Ends against the middle: determining public service provision when there are private alternatives. *Journal of Public Economics*, 62 (3), pp. 297-325.

Epple, D. & Romano, R. E. (1996b). Public provision of private goods. *Journal of Political Economy*, pp. 57-84.

Eurodyce (2008). A Governança do Ensino Superior na Europa. Gabinete de Estatística e Planeamento da Educação. Available at: [https://www.dgeec.mec.pt/np4/np4/%7B\\$clientServletPath%7D/?newsId=192&fileName=governanca_es_europa.pdf](https://www.dgeec.mec.pt/np4/np4/%7B$clientServletPath%7D/?newsId=192&fileName=governanca_es_europa.pdf).

European Commission (2021). Sobre a política de ensino superior. Comissão europeia. Available at: https://ec.europa.eu/education/policies/higher-education/about-higher-education-policy_pt.

Ezcurra, R. & Rodríguez-Pose, A. (2009). Decentralization of social protection expenditure and economic growth in the OECD. *Publius*, 41(1), pp. 146-57.

- Faggian, A. & McCann, P. (2009). *Human capital and regional development*. In: Capello R, Nijkamp P, editors. *Handbook of regional growth and development theories*. Cheltenham, England: Edward Elgar Publishing, pp. 133–51.
- Forman, C. & Zeebroeck, N. V. (2012). From wires to partners: how the internet has fostered R&D collaborations within firms. *Management Science*, 58 (8), pp. 1549–1568.
- Gamlath, S. & Lahiri, R. (2018). Public and private education expenditures, variable elasticity of substitution and economic growth. *Economic Modelling*, 70, pp. 1-14.
- Glomm, G. & Ravikumar, B. (1992). Public versus private investment in human capital: endogenous growth and income inequality. *Journal of Political Economy*, pp. 818–834.
- Glomm, G. (1997). Parental choice of human capital investment. *Journal of Development Economics*, 53 (1), pp. 99-114.
- Goldhaber, D. (1999). An endogenous model of public-school expenditures and private school enrollment. *Journal of Urban Economics*, 46 (1), pp. 106–128.
- Gouveia, M. (1997). Majority rule and the public provision of a private good. *Public choice*, 93 (3–4), pp. 221–244.
- Grossman, G. M. & Helpman, E. (1989). Comparative advantage and long-run growth. *NBER*, working Paper N° 2809.
- Grossman, G. M. & Helpman, E. (1991). Quality ladders in the theory of growth. *Review of Economic Studies*, 58 (1), pp. 43-61.
- Harrod, R. F. (1939). An Essay in Dynamic Theory. *Economic Journal*, 49 (193), pp. 14–33.
- Howitt, P. (1999). Steady endogenous growth with population and R. & D. inputs growing. *Journal of Political Economy*, 107(4), pp. 715-730.
- Iammarino, S., Rodriguez-Pose, A. & Storper, M. (2019). Regional inequality in Europe: evidence, theory and policy implications. *Journal of Economic Geography*, 19(2), pp. 273-298.
- Iberdrola (2021). Quais são os países mais inovadores do mundo?. Available at: <https://www.iberdrola.com/inovacao/paises-mais-inovadores-do-mundo>.
- International higher education (2021). International higher education. Center of International higher education, ed. 103. Available at: https://www.semesp.org.br/wp-content/uploads/2021/02/IHE_Boston_103_PORT.pdf.
- Jones, C. I. (1995a). R&D-based models of economic growth. *Journal of Political Economy*, 103(4), pp. 759–784.
- Jones, C. I. (1995b). Time Series Tests of Endogenous Growth Models. *The Quarterly Journal of Economics*, 110(2), pp. 495–525.
- Kortum, S. S. (1997). Research, Patenting, and Technological Change. *Econometrica*, 65(6), pp. 1389–1420.

- Lichtenberg, F. R. (1992). R&D investment and international productivity differences. *NBER*, Working Paper N. 4161.
- Mankiw, N. G., Romer, D. & Weil, D. A. (1992). Contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, 107, pp. 407–437.
- Moura, R. G. T. & Cruz, H.N. (2013). Teoria do crescimento endógeno e a inovação tecnológica no Brasil. *Revista de Administração e Inovação*, 10(3), pp. 230-250.
- Naidu, P. & Deran, N. E. S. (2016). A Comparative Study on Quality of Education Received by Students of Private Universities versus Public Universities. *Procedia Economics and Finance*, 35, pp. 659 – 666.
- Nair, M., Pradhan, R. P. & Arvin, M. B. (2020). Endogenous dynamics between R&D, ICT and economic growth: Empirical evidence from the OECD countries. *Technology in Society*, 62, PP. 101-315.
- Neves, P. C. & Sequeira, T. N. (2018). Spillovers in the production of knowledge: A meta-regression analysis. *Research Policy*, 47, pp. 750-767.
- Penn World Tables (2021). International comparisons of production, income and prices. Available at: <https://febpwt.webhosting.rug.nl/Dmn/AggregateXs/PivotShow#>.
- Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, 3(4), pp. 283-311.
- Peretto, P. F. (2018). Robust endogenous growth. *European Economic Review*, 108, pp. 49 – 77.
- Perotti, R. (1992). Income distribution, politics, and growth. *American Economic Review*, 82 (2), pp. 311–316.
- Perotti, R. (1996). Growth, income redistribution, and democracy: what the data say. *Journal of Economic Growth*, 1, pp. 149–87.
- Persson, T & Tabellini, G. (1992). Growth, distribution and politics. in Cukierman, A., Hercowitz Z., Leiderman, L. (eds): *Political Economy, Growth, and Business Cycles*, The MIT Press (Part I), Cambridge.
- Qs Top Universities (2021). QS World University Rankings 2021. Available at: <https://www.topuniversities.com/university-rankings/world-university-rankings/2021>.
- Romer, P. M. (1986). Increasing Returns and Long-Run Growth. *The Journal of Political Economy*, 94(5), pp. 1002-1037.
- Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5), pp. 71–102.
- Romer, P. M. (1994). The origins of endogenous growth. *Journal of Economic Perspectives*, 8 (1), pp. 3-22.

Sayer, S. (2000). Issues in new political economy: An overview. *Journal of Economic Surveys*, 14 (5), pp. 513–26.

Schneider, A., Horta, F. & Ioris, R. R. (2018, 24 January). A importância do Estado para o ensino superior dos EUA. Vermelho. Available at: <https://vermelho.org.br/2018/01/24/a-importancia-do-estado-para-o-ensino-superior-dos-eua/>.

Schumpeter, J. A. (1942). Capitalism, Socialism, and Democracy. *Harper*, Nova Iorque, NY.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *American Economic Review*, 88(5), pp. 1290–1310.

Segerstrom, P. S., Thirumalai, C. A. & Elias, D. (1990). A Schumpeterian model of the product life cycle. *American Economic Review*, pp. 1077-1091.

Sepehrdoust, H. & Zamani, S. S. (2015). Impact of knowledge-based components on total factor productivity of MENA countries, Iran. *Economic Review*, 19 (2), pp. 149–163.

Sianesi, B. & Van Reenen, J. (2003). The returns to education: Macroeconomics. *Journal of economic surveys*, 17(2), pp. 157–200.

Sochirca, E., Afonso, O. & Silva, S. T. (2017). Political rivalry effects on human capital accumulation and inequality: a new political economy approach. *Metroeconomica*, 68 (4), pp. 699-729.

Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1), pp. 65–94.

Solow, R. M. (1957). Technical Change and the Aggregate Production Function. *Review of Economics and Statistics*, 39 (3), 312-320.

Swan, T. W. (1956). Economic Growth and capital accumulation. *Economic Record*, 32(2), pp. 334-361.

Tang, S. (2012). Academic quality characteristics and satisfaction: An empirical survey among the students of two Malaysian private universities. *Academic Research International*, 2.

US News (2021, May 18). Top 10 Countries for Technological Expertise, Ranked by Perception: Countries with Tech Expertise, Ranked by Perception. Available at: <https://www.usnews.com/news/best-countries/slideshows/top-10-countries-for-technological-expertise-ranked-by-perception>.

Verstraete, T. (2002). *Essay on the Singularity of Entrepreneurship as a Research Domain*. Éditions de l'ADREG.

Walter, W., Powell, W. W. & Snellman, K. (2014). *The Knowledge Economy*. Stanford University Publication.

World Bank (2021). World Development Indicators. Available at: <https://databank.worldbank.org/source/world-development-indicators#>.

Young, A. (1998). Growth without scale effects. *Journal of Political Economy*, 106(1), pp. 41-63.